

Part III

Fourier and Related Integral Transforms

Integral transforms traditionally refer to the generalized expansion—as an *integral* rather than a series sum—of a function in a continuum of oscillating exponential or related functions. Of these, the prime example is the Fourier transform discussed in Chapter 7; other commonly used integral transforms, those associated with the names of Laplace, Mellin, and Hankel, are studied in Chapter 8. Applications are interspersed with the study of their relevant properties.

The presentation of the Fourier transform starts with the classic Fourier integral theorem and a survey of the relevant function spaces. The main properties under transformations and differential operators are then explored. With the introduction of the Dirac δ and the related task of finding the Green's function for an evolution equation, the basics are given for more specialized applications in the last three sections. These include causality and its description, oscillator wave-function bases including coherent states, and uncertainty relations. The last two, in addition to their inherent mathematical interest, are ubiquitous in quantum mechanics.

In Chapter 8 integral transforms related to Fourier transforms are applied to the description of unbounded diffusive and elastic media in one and more dimensions. The closing section is intended to provide a panorama of other integral transforms which appear in various situations.

